CH 4 PROBABILITY

ANSWERS AND EXPLANATIONS

EXERCISE 1

1. (b) Reqd probability = $\frac{^5C_2}{^7C_2} = \frac{5\times4}{7\times6} = \frac{10}{21}$

2. (e) If the drawn ball is neither red nor green, then it must be blue, which can be picked in $^7C_1$ = 7 ways. One ball can be picked from the total (8 + 7 + 6 = 21) in $^{21}C_1$ = 21 ways.

   $\therefore$ Reqd probability $= \frac{7}{21} = \frac{1}{3}$

3. (b) $n(S) = {}^{12}c_3 = \frac{12\times11\times10}{3\times2} = 2\times11\times10 = 220$

   No. of selection of 3 oranges out of the total 12 oranges =

   ${}^{12}c_3 = 2\times11\times10 = 220$.

   No. of selection of 3 bad oranges out of the total 4 bad oranges = $^4c_3$ = 4

   $\therefore$ $n(E)$ = no. of desired selection of oranges

   = 220 – 4 = 216

   $\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{216}{220} = \frac{54}{55}$

4. (b) Total no. of ways of drawing 3 marbles

   $= {}^{12}C_3 = \frac{12\times11\times10}{1\times2\times3} = 220$

   Total no. of ways of drawing marbles, which are of same colour

   = $^5C_3 + {}^4C_3 + {}^3C_3$

   = 10 + 4 + 1 = 15

   $\therefore$ Probability of same colour = $\frac{15}{220} = \frac{3}{44}$

   $\therefore$ Probalitity of not same colour = $1 - \frac{3}{44} = \frac{41}{42}$

5. (a) When two are thrown then there are 6 × 6 exhaustive cases \ n = 36. Let A denote the event "total score of 7" when 2 dice are thrown then A = [(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)].

   Thus there are 6 favourable cases.

   $\therefore$ m = 6 By definition $P(A) = \frac{m}{n}$

   $\therefore P(A) = \frac{6}{36} = \frac{1}{6}$.

6. (c) There are 5 + 7 = 12 balls in the bag and out of these two balls can be drawn in $^{12}C_2$ ways. There are 5 green balls, therefore, one green ball can be drawn in $^5C_1$ ways; similarly, one red ball can be drawn in $^7C_1$ ways so that the number of ways in which we can draw one green ball and the other red is $^5C_1 \times {}^7C_1$.

   Hence, P (one green and the other red)

   $= \frac{^5C_1\times{}^7C_1}{^{12}C_2} = \frac{5}{1}\times\frac{7}{1}\times\frac{1.2}{12.11} = \frac{35}{66}$.

7. (b) There are 7 + 5 = 12 balls in the bag and the number of ways in which 4 balls can be drawn is $^{12}C_4$ and the number of ways of drawing 4 black balls (out of seven) is $^7C_4$.

   Hence, P (4 black balls)

   $= \frac{^7C_4}{^{12}C_4} = \frac{7.6.5.4}{1.2.3.4}\times\frac{1.2.3.4}{12.11.10.9} = \frac{7}{99}$

Thus the odds against the event 'all black balls' are

$(1-\frac{7}{99}):\frac{7}{99}$: i.e., $\frac{92}{99}:\frac{7}{99}$ or $92:7$.

8. (b) The word 'SOCIETY' contains seven distinct letters and they can be arranged at random in a row in $^7P_7$ ways, i.e. in 7! = 5040 ways.

Let us now consider those arrangements in which all the three vowels come together. So in this case we have to arrange four letters. S,C,T,Y and a pack of three vowels in a row which can be done in $^5P_5$ i.e. 5! = 120 ways.

Also, the three vowels in their pack can be arranged in $^3P_3$ i.e. 3! = 6 ways.

Hence, the number of arrangements in which the three vowels come together is 120 × 6 = 720

∴ The probability that the vowels come together

$$=\frac{720}{5040}=\frac{1}{7}$$

9. (c) Let W stand for the winning of a game and L for losing it. Then there are 4 mutually exclusive possibilities

(i) W, W, W (ii) W, W, L, W

(iii) W, L, W, W (iv) L, W, W, W.

[Note that case (i) includes both the cases whether he losses or wins the fourth game.]

By the given conditions of the question, the probabilities for (i), (ii), (iii) and (iv) respectively are

$$\frac{2}{3}.\frac{2}{3}.\frac{2}{3};\ \frac{2}{3}.\frac{2}{3}.\frac{1}{3}.\frac{1}{3};\ \frac{2}{3}.\frac{1}{3}.\frac{1}{3}.\frac{2}{3} \text{ and } \frac{1}{3}.\frac{1}{3}.\frac{2}{3}.\frac{2}{3}.$$

Hence the required probability

$$=\frac{8}{27}+\frac{4}{81}+\frac{4}{81}+\frac{4}{81}=\frac{36}{81}=\frac{4}{9}.$$

[∵ The probability of winning the game if previous game was also won is $\frac{2}{1+2}=\frac{2}{3}$ and the probability of winning the game if previous game was a loss is $\frac{1}{1+2}=\frac{1}{3}$].

10. (b) The number of ways of choosing three numbers out of N is $^NC_3$. If these numbers are a_1, a_2 and a_3, they must satisfy exactly one of the following inequalities for a successful outcome.

$a_1<a_2<a_3$, $a_1<a_3<a_2$, $a_2<a_1<a_3$,

$a_2<a_3<a_1$, $a_3<a_1<a_2$, $a_3<a_2<a_1$.

Thus the number of ways of arranging the three numbers in a given order is $(^NC_3)$ (6), and there are 3 ways in which the first number is less than the second. Now if A denotes the event : the first number is less than the second number, and B the event : the third number lies between the first and the second, we need to find P(B|A). Since

$$P(B\cap A)=\frac{^NC_3}{(^NC_3)(6)}=\frac{1}{6} \text{ and } P(A)=\frac{(^NC_3)(3)}{(^NC_3)(6)}=\frac{1}{2},$$

We get $P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{1/6}{1/2}=\frac{1}{3}.$

11. (b) Suppose E_1, E_2 and E_3 are the events of winning the race by the horses A, B and C respectively

$$\therefore P(E_1)=\frac{1}{1+3}=\frac{1}{4},\quad P(E_2)=\frac{1}{1+4}=\frac{1}{5}$$

$$P(E_3)=\frac{1}{1+5}=\frac{1}{6}$$

∴ Probability of winning the race by one of the horses A, B and C

$$=P(E_1 \text{ or } E_2 \text{ or } E_3)=P(E_1)+P(E_2)+P(E_3)$$

$$=\frac{1}{4}+\frac{1}{5}+\frac{1}{6}=\frac{37}{60}$$

12. (d) Probability that only husband is selected

$$=P(H)P(\overline{W})=\frac{1}{7}\left(1-\frac{1}{5}\right)=\frac{1}{7}\times\frac{4}{5}=\frac{4}{35}$$

Probability that only wife is selected

$= P(\overline{H})P(W) = \left(1-\frac{1}{7}\right)\left(\frac{1}{5}\right) = \frac{6}{7}\times\frac{1}{5} = \frac{6}{35}$

$\therefore$ Probability that only one of them is selected

$= \frac{4}{35}+\frac{6}{35}=\frac{10}{35}=\frac{2}{7}$

13. (c) Probability of selecting a month $= \frac{1}{12}$.

13^{th} day of the month is friday if its first day is sunday and the probability of this $= \frac{1}{7}$.

$\therefore$ Required probability $= \frac{1}{12}.\frac{1}{7}=\frac{1}{84}$.

14. (c) A leap-year has 366 days i.e. 52 complete weeks and two days more these two days be two consecutive days of a week. A leap year will have 53 Sundays if out of the two consecutive days of a week selected at random one is a Sunday.

Let S be the sample space and E be the event that out of the two consecutive days of a week one is Sunday, then

S = {(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)}

$\therefore$ n (S) = 7

and E = {(Sunday, Monday), (Saturday, Sunday)

$\therefore$ n (E) = 2

Now, required Probability, $P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$

15. (c) Numbers divisible by 4 are 104, 108.., 196; 24 in number. Numbers divisible by 7 are 105, 112, ...196; 14 in number. Numbers divisible by both, i.e.divisible by 28 are 112, 140, 168, 196; 4 in number. Hence, required probability

$= \frac{24}{99}+\frac{14}{99}-\frac{4}{99}=\frac{34}{99}$

16. (d) Total of seven can be obtained in the following ways

1, 1, 1, 4 in $\frac{4!}{3!}$ = 4 ways

[there are four objects, three repeated]

Similarly,

1, 1,2, 3 in $\frac{4!}{2!}$ = 12 ways

1, 2,2, 2 in $\frac{4!}{3!}$ = 4 ways

Hence, required probability

$= \frac{4+12+4}{6^4}=\frac{20}{6^4}$

[$\because$ Exhaustive no. of cases = $6\times6\times6\times6 = 6^4$]

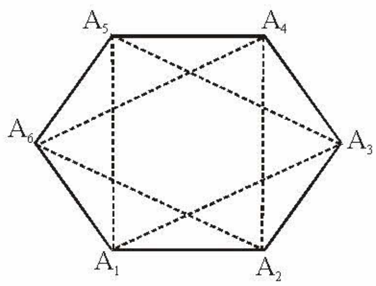
17. (b) Since, A and B are independent events $\therefore$ A' and B' are also independent events

$\Rightarrow P(A'\cap B') = P(A').P(B')$

$= (0.4)(0.7) = 0.28$

$[\because P(A') = 1-P(A),\ P(B') = 1-P(B)]$

18. (c) Three vertices can be selected in ${}^6C_3$ ways.



The only equilateral triangles possible are $A_1A_3A_5$ and $A_2A_4A_6$

$p = \frac{2}{{}^6C_3}=\frac{2}{20}=\frac{1}{10}$

19. (c) The number of ways of getting the different number 1, 2, ..., 6 in six dice = 6 !.

Total number of ways = 6^6

Hence, required probability = $\frac{6!}{6^6}$

$= \frac{1\times2\times3\times4\times5\times6}{6^6} = \frac{5}{324}$

20. (a) Total number of balls = 12

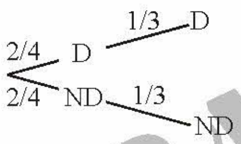
Hence, required probability

$= \frac{^5C_2 \times {}^7C_2}{^{12}C_4} = \frac{14}{33}$

$\because$ No of ways of drawing 2 white balls from 5 white balls = $^5C_2$.

Also, No of ways of drawing 2 other from remaining 7 balls = $^7C_2$

21. (a) The faulty machines can be identified in two tests only if both the tested machines are either all defective or all non-defective. See the following tree diagram.



(Here D is for Defective & ND is for Non Defective)

Reqd. Probability = $\frac{2}{4}\times\frac{1}{3}+\frac{2}{4}\times\frac{1}{3}=\frac{1}{3}$

$\because$ The probability that first machine is defective (or non-defective) is $\frac{2}{4}$ and the probability that second machine is also defective (or non - defective) is $\frac{1}{3}$ as 1 defective machine remains in total three machines.

22. (b) The probality that the person hits the target = 0.3

$\therefore$ The probability that he does not hit the target in a trial = 1 – 0.3 = 0.7

$\therefore$ The probability that he does not hit the target in any of the ten trials = $(0.7)^{10}$

$\therefore$ Probability that he hits the target

= Probability that at least one of the trials succeeds

= $1 - (0.7)^{10}$.

23. (c) We have $P(A\cup B)=0.7$ and $P(A\cap B)=0.2$

Now, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A)+P(B)=0.9 \Rightarrow 1-P(\overline{A})+1-P(\overline{B})=0.9$

$\Rightarrow P(\overline{A})+P(\overline{B})=1.1$

24. (b) Let n tosses be required.

The probability of getting a head in any toss = $\frac{1}{2}$

The probability of not getting a head in a toss = $\frac{1}{2}$

The probability of not getting any head in n tosses $=\left(\frac{1}{2}\right)^n$. Hence probability of getting at least one head in n tosses = $1 - (1/2)^n$. Therefore

$1-(1/2)^n \ge 0.9 \Rightarrow (1/2)^n \le 0.1$

$\Rightarrow 2^n \ge 10 \Rightarrow n \ge 4.$

Hence, the least value of n is 4.

25. (a) The probability that A cannot solve the problem $=1-\frac{2}{3}=\frac{1}{3}$

The probability that B cannot solve the problem

$=1-\frac{3}{4}=\frac{1}{4}$

The probability that both A and B cannot solve the problem $=\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$

$\therefore$ The probability that at least one of A and B can solve the problem $=1-\frac{1}{12}=\frac{11}{12}$

$\therefore$ The probability that the problem is solved = $\frac{11}{12}$

26. (b) Only one of A and B can be alive in the following, mutually exclusive ways.

E_1 A will die and B will live

E_2 B will die and A will live

So, required probability = $P(E_1) + P(E_2)$

$= p(1-q) + q(1-p) = p + q - 2pq.$

27. (c) The total number of ways in which 3 integers can be chosen from first 20 integers is $^{20}C_3$.

The product of three integers will be even if at least one of the integers is even. Therfore, the required probability = 1 – Prob. that none of the three integers is even

$$= 1 - \frac{^{10}C_3}{^{20}C_3} = 1 - \frac{2}{19} = \frac{17}{19}.$$

[Three odd integers can be chosen in $^{10}C_3$ ways as there are 10 even and 10 odd integers.

28. (a) If a probability p is assigned to each even number, then 2p is the probability to be assigned to each odd number which gives $2p \times 3 + p \times 3 = 9p = 1$.

$$\Rightarrow p = \frac{1}{9}$$

$\therefore$ P(E) = Probability of getting 4, 5 or 6

$$= \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$$

29. (b) The condition implies that the last digit in both the integers should be 0, 1, 5 or 6 and the probability

$$= 4\left(\frac{1}{10}\right)^2 = \frac{4}{100} = \frac{1}{25}$$

[$\because$ The squares of numbers ending in 0 or 1 or 5 or 6 also 0 or 1 or 5 or 6 respectively]

30. (a) Seven people can seat themselves at a round table in 6! ways. The number of ways in which two distinguished persons will be next to each other = 2 (5) !, Hence, the required probability

$$= \frac{2(5)!}{6!} = \frac{1}{3}$$

EXERCISE 2

1. (a) $\therefore S = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

$\therefore n(S) = 36$

& Let $E_1 \equiv$ the event that the sum of the numbers coming up is 9.

& $E_2 \equiv$ the event of occurrence of 5 on the first die.

$E_1 \equiv \{(3, 6), (6, 3), (4, 5), (5, 4)\}$

$\therefore n(E_1) = 4$ and

$E_2 = \{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

$\therefore n(E_2) = 6$

$E_1 \cap E_2 = \{(5,4)\}$

$\therefore n(E_1 \cap E_2) = 1$

Now, $P(E_1 \cap E_2) = \dfrac{n(E_1 \cap E_2)}{n(S)} = \dfrac{1}{36}$

and $P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

$\therefore$ Required Probability

$$= P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$$

2. (c) Two squares out of 64 can be selected in

$$^{64}C_2 = \frac{64 \times 63}{2} = 32 \times 63 \text{ ways}$$

The number of ways of selecting those pairs which have a side in common

$$= \left(\frac{1}{2}\right)(4\times2 + 24\times3 + 36\times4) = 112$$

[Since each of the corner squares has two neighbours each of 24 squares in border rows, other than corner ones has three neighbours and each of the remaining 36 squares have four neighours and in this computation, each pair of squares has been considered twice].

Hence required probability $= \frac{.112}{32 \times 63} = \frac{1}{18}$.

3. (b) A and B will contradict each other if one of the events $A \cap B'$ or $A' \cap B$ occurs. The probability of this happening is

$$P[(A \cap B') \cup (A' \cap B) = P(A \cap B') + P(A' \cap B)$$

$$= P(A)P(B') + P(A)P(B),$$

because A and B are independent. Therefore, putting P(A) = 0.7 and P(B) = 0.8 the required probability is (0.7) (0.2) + (0.3) (0.8) = 0.38.

4. (d) Let E_1 be the event of drawing a card printed I in the first draw.

E_2 be the event of drawing a card printed I in the second draw and E_3 be the event of drawing a card printed T in the third draw.

then, the required probability is

$$P(E_1 \cap E_2 \cap E_3) = P(E_1).P(E_2 / E_1).P(E_3 / E_1 \cap E_2)$$

$$= \frac{10}{20}.\frac{9}{19}.\frac{10}{18} = \frac{5}{38}$$

5. (c) We have (S = 9) = {09, 18, 27, 36, 45, 54, 63, 72, 81, 90} and (T = 0) = {0, 01,.......,09, 10, 20, ...90}

Also (S = 9) $\cap$ (T = 0) = {09, 90}

Thus P((S = 9) $\cap$ (T = 0)) = 2 / 100

Hence P (S = 9/T = 0)

$$= \frac{P((S = 9) \cap (T = 0))}{P(T = 0)} = \frac{2/100}{19/100} = \frac{2}{19}.$$

6. (b) $p_1 = \frac{^{15}C_2}{^{42}C_4} = \frac{15 \times 14 \times 4!}{2! \times 42 \times 41 \times 40 \times 39} = \frac{1}{41 \times 26}$ and

$$p_2 = \frac{^{30}C_4}{^{84}C_8} = \frac{15 \times 14 \times 13 \times 12 \times 8!}{4! \times 84 \times 83 \times 82 \times ..... \times 77}$$

$$= \frac{15 \times 14 \times 13 \times 12 \times 8 \times 7 \times 6 \times 5}{84 \times 83 \times 82 \times 81 \times 79 \times 78 \times 77} < p_1$$

$\Rightarrow p_1 > p_2$.

7. (a) $\frac{1+4p}{4}, \frac{1-p}{2}, \frac{1-2p}{2}$ are probabilities of the three mutually exclusive events, then

$$0 \le \frac{1+4p}{4} \le 1,\ 0 \le \frac{1-p}{2} \le 1,\ 0 \le \frac{1-2p}{2} \le 1$$

and $0 \le \frac{1+4p}{4} + \frac{1-p}{2} + \frac{1-2p}{2} \le 1$

$$\therefore -\frac{1}{4} \le p \le \frac{3}{4}, -1 \le p \le 1, -\frac{1}{2} \le p \frac{1}{2}, \frac{1}{2} \le p \le \frac{5}{2}$$

$$\therefore \frac{1}{2} \le p \le \frac{1}{2}$$

[The intersection of above four intervals]

$$\therefore p = \frac{1}{2}$$

8. (a) Let A, B and C be the events that the student is successful in tests I, II and III respectively.

Then P (The student is successful)

$$= P(A)P(B)\{1 - P(C)\} + P(A)\{1 - P(B)\}P(C) +$$

$$P(A)P(B)P(C)$$

$$= p\ .\ q\left(1 - \frac{1}{2}\right) + p(1 - q)\ \frac{1}{2} + p\ .\ q\ \frac{1}{2}$$

$$= \frac{1}{2}pq + \frac{1}{2}p(1-q) + \frac{1}{2}pq$$

$$= \frac{1}{2}(pq + p - pq + pq) = \frac{1}{2}(pq + p)$$

$$\therefore \frac{1}{2} = \frac{1}{2}(pq + p) \Rightarrow 1 = pq + p$$

9. (a) Since $P(A \cup B \cup C) \ge 0.75$, therefore

$$0.75 \le P(A \cup B \cup C) \le 1$$

$$\Rightarrow 0.75 \le P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) -$$

$$P(A \cap C) + P(A \cap B \cap C) \le 1$$

$$\Rightarrow 0.75 \le 0.3 + 0.4 + 0.8 - 0.08 - P(B \cap C) -$$

$$0.28 + 0.09 \le 1$$

$\Rightarrow 0.75 \le 1.23 - P(B \cap C) \le 1$

$\Rightarrow -0.48 \le -P(B \cap C) \le -0.23$

$\Rightarrow 0.23 \le P(B \cap C) \le 0.48.$

10. (d) $P(i) = \frac{k}{i}$

$\Rightarrow 1 = \sum_{i=1}^{6} P(i) = k\sum_{i=1}^{6} \frac{1}{i} = k\frac{49}{20} \Rightarrow k = \frac{20}{49}$

$\therefore P(3) = \frac{k}{3} = \frac{20}{49 \times 3} = \frac{20}{147}$

11. (c) Probability of A winning [A can win in 1^{st} or 3^{rd} or 5^{th}.... games if B loses 2nd or 4th or... games]

$= \frac{p}{p+q} + \left(\frac{q}{p+q}\right)^2 . \frac{p}{p+q} + \left(\frac{q}{p+q}\right)^4 . \frac{p}{p+q} + ...$

$= \frac{\frac{p}{p+q}}{1-\left(\frac{q}{p+q}\right)^2} \left[\text{In infinite G.P. } S = \frac{a}{1-r}\right]$

$= \frac{p(p+q)}{(p+q)^2 - q^2}$

Probability of B winning

$= 1 - \frac{p(p+q)}{(p+q)^2 - q^2} = \frac{(p+q)^2 - q^2 - p(p+q)}{(p+q)^2 - q^2}$

Given, P (A) = 3 P(B)

$\Rightarrow p(p+q) = 3[(p+q)^2 - q^2 - p(p+q)]$

$\Rightarrow 4p(p+q) = 3(p+2q).p$

$\Rightarrow 4p + 4q = 3p + 6q \Rightarrow p = 2q$

$\frac{p}{q} = 2$ or $p : q = 2 : 1$

12. (b) Exhaustive no. of cases = 6^3

10 can appear on three dice either as distinct number as following (1, 3, 6) ; (1, 4, 5); (2, 3, 5) and each can occur in 3! ways. Or 10 can appear on three dice as repeated digits as following (2, 2, 6), (2, 4, 4), (3, 3, 4) and each can occur in $\frac{3!}{2!}$ ways.

$\therefore$ No. of favourable cases $= 3 \times 3! + 3 \times \frac{3!}{2!} = 27$

Hence, the required probability $= \frac{27}{6^3} = \frac{1}{8}$

13. (b) Since each ball can be put into any one of the three boxes. So, the total number of ways in which 12 balls can be put into three boxes is 3^{12}.

Out of 12 balls, 3 balls can be chosen in ${}^{12}C_3$ ways. Now, remaining 9 balls can be put in the remaining 2 boxes in 2^9 ways. So, the total number or ways in which 3 balls are put in the first box and the remaining in other two boxes is ${}^{12}C_3 \times 2^9$.

Hence, required probability $= \frac{{}^{12}C_3 . 2^9}{3^{12}}$

14. (a) For each toss there are four choices

(i) A gets head, B gets head

(ii) A gets tail, B gets head

(iii) A gets head, B gets tail

(iv) A gets tail, B gets tail

thus, exhaustive number of ways = 4^{50}. Out of the four choices listed above (iv) is not favourable to the required event in a toss. Therefore favourable number of cases is 3^{50}.

Hence, the required probability $= \left(\frac{3}{4}\right)^{50}$

15. (a) Since, A and B are independent events.

$\therefore P(A \cap B) = P(A)P(B)$

Further since, $A \cap C, B \cap C, A \cap B \cap C$ are subsets of C, we have

$P(A \cap C) \le P(C) = 0$

$P(B \cap C) \le P(C) = 0$

and $P(A \cap B \cap C) \le P(C) = 0$

$\Rightarrow P(A \cap C) = 0 = P(A)P(C)$

$P(B \cap C) = 0 = P(B)P(C)$

$P(A \cap B \cap C) = 0 = P(A)P(B)P(C)$.

Clearly A, B, C are pairwise independent as well as mutually independent. Thus, A,B,C are independent events.

16. (d) The number of ways of arranging 50 books = ${}^{50}P_{50}$ = 50!. The number of ways of choosing places for the five volume dictionary is ${}^{50}C_5$ and the number of ways of arranging the remaining 45 books = ${}^{45}C_{45}$= (45)! Thus the number of favourable ways is (${}^{50}C_5$) (45 !). Hence the probability of the required event

$$= \frac{({}^{50}C_5)(45!)}{50!} = \left(\frac{50!}{5!\,45!}\right)\left(\frac{45!}{50!}\right) = \frac{1}{5!} = \frac{1}{120}$$

17. (b) Given equation

$$x + \frac{100}{x} > 50$$

$$\Rightarrow x^2 - 50x + 100 > 0$$

$$\Rightarrow (x-25)^2 > 525$$

$$\Rightarrow x - 25 < -\sqrt{(525)} \text{ or } x - 25 > \sqrt{(525)}$$

$$\Rightarrow x < 25 - \sqrt{(525)} \text{ or } x > 25 + \sqrt{(525)}$$

As x is positive integer and $\sqrt{(525)} = 22.91$, we must have

$x \le 2$ or $x \ge 48$

Let E be the event for favourable cases and S be the sample space.

$\therefore E = \{1, 2, 48, 49, ...100\}$

$\therefore n(E) = 55$ and $n(S) = 100$

Hence the required probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{55}{100} = \frac{11}{20}.$$

18. (d) There are three mutually and exhaustive ways in which 2 balls can be transferred from first bag to second bag.

(1) The balls transferred are both white its probability is $\frac{{}^4C_2}{{}^6C_2} = \frac{6}{16} = \frac{2}{5}$.

The second bag then contains 7 white and 4 black balls. The probability of drawing a white ball is $\frac{7}{11}$.

$\therefore$ The combined probability of both the steps

$$= \frac{2}{5} \times \frac{7}{11} = \frac{14}{55}$$

(2) The balls transferred are both black its probability is $\frac{{}^2C_2}{{}^6C_2} = \frac{1}{15}$

The second bag then contains 5 white and 6 black balls. The probability of drawing a white ball is $\frac{5}{11}$.

$\therefore$ The combined probability of both the steps

$$= \frac{1}{15} \times \frac{5}{11} = \frac{1}{33}$$

(3) One black and one white balls are transferred. Its probability is $\frac{{}^4C_1 \times {}^2C_1}{{}^6C_2} = \frac{8}{15}$

The second bag then contains 6 white and 5 black balls. The probability of drawing a white ball is $\frac{6}{11}$.

$\therefore$ The combined probability of both the steps

$$= \frac{8}{15} \times \frac{6}{11} = \frac{16}{55}$$

The required probability of drawing a white ball from the second bag

$$= \frac{14}{55} + \frac{1}{33} + \frac{16}{55} = \frac{95}{165} = \frac{19}{33}.$$

19. (a) In any number the last digit can be one of 0, 1, 2, ... 8, 9. Therefore, the last digit of each number can be chosen in 10 ways. Thus, exhausitive number of ways = 10^n. If the last digit be 1, 3, 7 or 9 none of the numbers can be even or end in 0 or 5. Thus, we have a choice of 4 digits viz. 1, 3, 7 or 9 with which each of n number should end. So, favourable number of ways = 4^n. Hence, the

required probability = $\frac{4^n}{10^n} = \left(\frac{2}{5}\right)^n$.

20. (a) Total number of attempts = 20

Favourable no. of attempts = 5

Required probability (running the program correctly in the third run) $= \frac{5}{20} = \frac{1}{4}$

21. (b) Probability (sending a correct programme)

$= 1 - \frac{1}{5} = \frac{4}{5}$

Probability (the packet is not damaged)

$= 1 - \frac{3}{4} = \frac{1}{4}$

Probability (there is no short shipment)

$= 1 - \frac{1}{3} = \frac{2}{3}$

Required probability

$= \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} = \frac{2}{15} = \frac{8}{60}$

22. (c) Probability of occurence of head in a toss of a coin is 1/2.

Required probability = Prob[Head appears once] + Prob.[Head appears thrice] + Prob.[Head appears five times]

$$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 [5 + 10 + 1] = \frac{16}{32} = \frac{1}{2}$$

23. (b) Chandra hits the target 4 times in 4 shots. Hence, he hits the target definitely.

The required probability, therefore, is given by.

P(both Atul and Bhola hit) + P(Atul hits, Bhola does not hit) + P(Atul does not hit, Bhola hits)

$$= \frac{3}{6} \times \frac{2}{6} + \frac{3}{6} \times \frac{4}{6} + \frac{3}{6} \times \frac{2}{6}$$

$$= \frac{1}{6} + \frac{1}{3} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

24. (a) Total number of balls = 5 + 7 + 8 = 20

Probability that the first ball drawn is white

$= \frac{{}^5C_1}{{}^{20}C_1} = \frac{1}{4}$

If balls are drawn with replacement, all the four events will have equal probability.

Therefore, required probability

$= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{256}$

25. (b) Let A be the event of getting an odd number.

Here, n (S) = 6 and

n (A) = 3

Probability of getting an odd number

$= \frac{3}{6} = \frac{1}{2}$

Hence, probability of not getting an odd number

$= 1 - \frac{1}{2} = \frac{1}{2}$

Required probability of 5 successes

$$= {}^6C_5 \times \left(\frac{1}{2}\right)^5 \times \frac{1}{2} = \frac{3}{32}$$

26. (c) Required probability

= Probability that ball from bag A is red and both the balls from bag B are black + Probability that ball from bag A is black and one black and one red balls are drawn from bag B

$$= \frac{{}^4C_1}{{}^9C_1} \times \frac{{}^7C_2}{{}^{10}C_2} + \frac{{}^5C_1}{{}^9C_1} \times \frac{{}^3C_1 \times {}^7C_1}{{}^{10}C_2}$$

$= \frac{4}{9} \times \frac{7}{15} + \frac{5}{9} \times \frac{7}{15} = \frac{7}{15}$

27. (c) n (S) = 4 × 3 × 2 × 1   O O O O

n (A) = 3 × 2 × 2 × 1   1 2 3 4

∴ Required probability = $\frac{12}{24} = \frac{1}{2}$

28. (c) Total no. of numbers = 6 positive + 8 negative = 14

$n(S) = {}^{14}C_4$

The product of four numbers could be positive when,

(a) all the four numbers chosen are positive or

(b) all the four numbers chosen are negative or

(c) two of the chosen numbers are positive and two are negative.

$$\text{Required Prob.} = \frac{{}^6C_4}{{}^{14}C_4} + \frac{{}^8C_4}{{}^{14}C_4} + \frac{{}^6C_2 \times {}^8C_2}{{}^{14}C_4}$$

$$= \frac{505}{1001}$$

29. (b) Total no. of outcomes when two dice are thrown = n (S) = 36 and the possible cases for the event that the sum of numbers on two dice is a prime number, are

(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 1), (5, 6), (6, 1), (6, 5)

Number of outcomes favouring the event = n (A) = 15

Required probability $= \frac{n(A)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

30. (d) The probability of selecting a bag $= \frac{1}{2}$

Now, probability of getting a white ball from the first bag $= \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

and probability of getting a white ball from the second bag $= \frac{1}{2} \times \frac{2}{6} = \frac{1}{6}$

Required Probability = The probability that a white ball is drawn either from the first or the second bag $= \frac{3}{10} + \frac{1}{6} = \frac{7}{15}$

31. (d) Let X be the number of heads

Now, P (X = 50) = P (X = 51)

or $\;{}^{100}C_{50} \times p^{50} \; (1 - p)^{50}$

$= {}^{100}C_{51} \times p^{51} \; (1 - p)^{49}$

or $\;{}^{100}C_{50} \times (1 - p) = {}^{100}C_{51} \times p$

or $\;51 \, (1 - p) = 50 \, p$

or $\;p = \frac{51}{101}$

32. (c) If six coins are tossed, then the total no. of outcomes $= (2)^6 = 64$

Now, probability of getting no tail $= \frac{1}{64}$

Probability of getting at least one tail

$= 1 - \frac{1}{64} = \frac{63}{64}$

33. (c) 4 students out of 5 can be selected in ${}^5C_4$ ways.

Probability of a student being not a swimmer $= \frac{1}{5}$

Probability of a student being a swimmer

$= \left(1 - \frac{1}{5}\right) = \frac{4}{5}$

Required probability $= {}^5C_4 \times \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$

34. (b) 2 balls can be drawn in the following ways

1 red and 1 green or 2 red or 2 green

$$\text{Required probability} = \frac{{}^2C_1 \times {}^3C_1}{{}^7C_2} + \frac{{}^2C_2}{{}^7C_2} + \frac{{}^3C_2}{{}^7C_2}$$

$$= \frac{6}{21} + \frac{1}{21} + \frac{3}{21} = \frac{10}{21}$$

35. (d) It is given that last 3 digits are randomly dialled. Then each of the digits can be selected out of 10

digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) in 10 ways.

Hence the required probability $=\left(\frac{1}{10}\right)^3=\frac{1}{1000}$

36. (d) Probability that the trouser is not black $=\frac{2}{3}$

Probability that the shirt is not black $=\frac{3}{4}$

Since, picking of a shirt and a trouser are independent,

required probability $=\frac{2}{3}\times\frac{3}{4}=\frac{1}{2}$

37. (c) Exhaustive number of cases $= {}^{24}C_{14}$

Favourable cases $= {}^{22}C_{14}$

38. (a) The event that the fifth toss results a head is independent of the event that the first four tosses results tails.

$\therefore$ Probability of the required event = 1/2

39. (c) $7^m + 7^n = [(5+2)^m + (5+2)^n]$

$\equiv 5 \times$ integer $+ 2^m + 2^n$

$\therefore 7^m + 7^n$ is divisible by 5 iff $2^m + 2^n$ is divisible by 5 and so unit place of $2^m + 2^n$ must be 0 since it cannot be 5.

m	possible n		
1	3,7, 11,15, ...	=	25
2	4, 8,12,.....	=	25
3	1, 5, 9, ...	=	25
4	2, 6,10, ...	=	25

Since $2^1 + 2^3 \equiv 2^3 + 2^1$. So (1, 3) and (3, 1) are same as favourable cases.

$\therefore$ Required probability $=\frac{25\times 50}{100\times 100}=\frac{1}{8}$

40. (d) Exhaustive number of cases = 12

Favourable cases $= {}^{12}C_2\,(2^6-2)$

$\therefore$ Probabiltity $=\frac{{}^{12}C_2(2^6-2)}{12^6}=\frac{341}{12^5}$

41. (a) $P(E_1)=\frac{1}{2}$, $P(E_2)=\frac{1}{3}$ and $P(E_3)=\frac{1}{4}$;

$$P(E_1 \cup E_2 \cup E_3) = 1 - P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3)$$

$$=1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{3}{4}$$

42. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;

$\Rightarrow \frac{3}{4} = 1 - P(\bar{A}) + P(B) - \frac{1}{4}$

$\Rightarrow 1 = 1 - \frac{2}{3} + P(B) \Rightarrow P(B) = \frac{2}{3}$;

Now, $P(\bar{A} \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$.

43. (a) $n(S) = {}^5C_2$; $n(E) = {}^2C_1 + {}^2C_1$

$P(E) = \frac{n(E)}{n(S)} = \frac{{}^2C_1 + {}^2C_1}{{}^5C_2} = \frac{2}{5}$

44. (c) A and B will contradict each other if one speaks truth and other false . So, the required

Probability $=\frac{4}{5}\left(1-\frac{3}{4}\right)+\left(1-\frac{4}{5}\right)\frac{3}{4}$

$=\frac{4}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}=\frac{7}{20}$