CH 2 PIPES AND CISTERNS

ANSWERS AND EXPLANATIONS

EXERCISE 1

1. (c) Part of the tank filled in one hour

$= \frac{1}{8} - \frac{1}{16} = \frac{1}{16}$

Hence, the tank will be filled in 16 hours.

2. (d) Let the exhaust tap empties the tank in x minutes.

Then, $\frac{1}{12} + \frac{1}{15} - \frac{1}{x} = \frac{1}{20}$

or $\frac{1}{x} = \frac{1}{12} + \frac{1}{15} - \frac{1}{20}$

or $\frac{1}{x} = \frac{5+4-3}{60} = \frac{6}{60} = \frac{1}{10}$ or x = 10 min

3. (c) Let the leak empties the tank in x hours.

Now, $\frac{1}{5} - \frac{1}{x} = \frac{1}{6}$

or $\frac{1}{x} = \frac{1}{5} - \frac{1}{6} = \frac{1}{30}$

or x = 30 hrs.

4. (c) Let pipe A fills the cistern in x minutes.

Therefore, pipe B will fill the cistern in (x + 5) minutes.

Now, $\frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$

$\Rightarrow$ x = 10

Thus, the pipes A and B can fill the cistern respectively in 10 minutes and 15 minutes,

5. (a) Portion of the tank filled by all the pipes together in 1 hour $= \frac{1}{10} + \frac{1}{12} - \frac{1}{20}$

$= \frac{6+5-3}{60} = \frac{8}{60} = \frac{2}{15}$

Hence, the tank will be filled in $\frac{15}{2}$ hours

or $7\frac{1}{2}$ hours.

6. (a) Part of the capacity of the cistern emptied by the leak in one hour

$= \left(\frac{1}{6} - \frac{1}{7}\right) = \frac{1}{42}$ of the cistern.

The whole cistern will be emptied in 42 hours.

7. (b) Work of both tap for 1 hour

$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Hence, both tap will fill the cistern in 6 hours.

8. (c) Part filled by (A + B + C) in 1 hour

$= \left(\frac{1}{5} + \frac{1}{10} + \frac{1}{30}\right) = \frac{1}{3}$.

$\therefore$ All the three pipes together will fill the tank in 3 hours.

9. (c) Work done by the waste pipe in 1 minute

$= \frac{1}{15} - \left(\frac{1}{20} + \frac{1}{24}\right) = \left(\frac{1}{15} - \frac{11}{120}\right) = -\frac{1}{40}$.

[–ve sign means emptying]

$\therefore$ Volume of $\frac{1}{40}$ part = 3 gallons.

Volume of whole = (3 × 40) gallons = 120 gallons.

10. (d) Capacity of the tank = (12 × 13.5) litres = 162 litres.

Capacity of each bucket = 9 litres.

Number of buckets needed

$= \left(\frac{162}{9}\right) = 18.$

11. (d) Tank filled in 1 minute

$= \frac{1}{25} + \frac{1}{40} - \frac{1}{30}$ part

$= \frac{24+15-20}{600} = \frac{19}{600}$ part

$\therefore$ tank will be filled complete in minutes

$= \frac{600}{19} = 31\frac{11}{19}$

12. (a) Here ratio of efficiencies of pipes A, B and C are as follows:

C		B		A
2		1		
		2		1
4	:	2	:	1

Suppose the efficiencies of pipes C, B and A are 4K, 2K and K.

Since, the tank is filled in 5 hours by the three pipes having combined efficiency equal to 7K, the time required to fill the tank by A alone

$= \frac{7K \times 5}{K} = 35$ hours

13. (a)

$\because$ Pipe *A* in 1 minute fills 1/10 part and Pipe *B* in 1 min. empties $\frac{1}{6}$ part

$\therefore$ Pipe A + B in 1 min $= \frac{1}{10} - \frac{1}{6} = \frac{-1}{15}$

$\because$ $\frac{1}{15}$ part gets emptied in 1 min

$\therefore$ $\frac{2}{5}$ part is emptied in $15 \times \frac{2}{5}$ min = 6 min

EXERCISE 2

1. (d) Capacity of water throwing pump

$= \frac{12}{60} \times 5.5 = 1.1$ tonnes per 5.5 minutes

Capacity of the leak to admit water

= 2.25 tonnes per 5.5 minutes

In 5.5 minutes, net water accumulated by the leak

= (2.25 – 1.1) = 1.15 tonnes

Thus, to admit 92 tonnes of water, it will take

$\frac{5.5}{1.15} \times 92$

= 440 min $= \frac{440}{60}$ hrs

Speed required for the ship to sail through safely

$= \frac{77 \times 60}{440} = 10.5 \text{km/h}$

2. (c) If both the pumps are opened together, then the tank will be emptied because the working efficiency of pump empting is more than that of the pump filling it. Thus in 1 min net work done

$= \left(\frac{1}{8} - \frac{1}{16}\right) = \frac{1}{16}$ parts

or the tank will be emptied in 16 min

$\Rightarrow$ $\frac{1}{2}$ tank will be emptied in 8 min.

3. (c) Proportion of the volume of the tank filled by both the pipes in 4 min

$= 4\left(\frac{1}{15} + \frac{1}{10}\right) = \frac{2}{3}$rd of the tank.

Volume of the tank filled by all the pipes working together

$= \frac{1}{15} + \frac{1}{10} - \frac{1}{5} = \frac{-1}{30}$

i.e. $\frac{1}{30}$ tank is emptied in 1 min.

4. (d) A + B fill in 6 hrs.

B + C fill in 10 hrs.

A + C fill in $7\frac{1}{2} = \frac{15}{2}$ hrs

$\therefore$ 2 (A + B + C) fill in

$$\frac{6\times10\times\frac{15}{2}}{6\times10+6\times\frac{15}{2}+10\times\frac{15}{2}}$$

$$= \frac{6\times5\times15}{180} = \frac{5}{2}$$

$\therefore$ A + B + C filled the tank in 5 hrs.

Now, A[= (A + B + C) – (B + C)] fill in

$$\frac{10\times5}{10-5} = 10\text{hrs.}$$

Similarly, B fill in

$$\frac{\frac{15}{2}\times5}{\frac{15}{2}-5} = 15 \text{ hrs and}$$

C fill in $\frac{5\times6}{6-5} = 30$ hrs.

5. (c) Net part filled in 1 hour

$$= \left(\frac{1}{5}+\frac{1}{6}-\frac{1}{12}\right)=\frac{17}{60}.$$

$\therefore$ The tank will be full in

$\frac{60}{17}$ hrs i.e., $3\frac{9}{17}$ hrs.

6. (d) Part filled by first tap in one min $= \frac{1}{12}$th

Part filled by second tap in one min $= \frac{1}{18}$th

Now, $2\left[\frac{1}{12}+\frac{1}{18}\right]$ + unfilled part = 1

$\Rightarrow$ unfilled part = $\frac{13}{18}$th

$\because$ $\frac{1}{18}$th part of tank is filled by second tap in 1min.

$\therefore$ $\frac{13}{18}$th part of tank is filled by second tap in 1 min.

$$= 18\times\frac{13}{18}\text{ min} = 13\text{ min.}$$

7. (b) In one min, (A + B) fill the cistern

$$= \frac{1}{10}+\frac{1}{15}=\frac{1}{6}\text{th}$$

In 3 min, (A + B) fill the cistern = $\frac{3}{6}=\frac{1}{2}$th

Remaining part $= 1-\frac{1}{2}=\frac{1}{2}$

$\because$ $\frac{1}{10}$th part filled by A in one min.

$\therefore$ $\frac{1}{2}$nd part filled by A in $10\times\frac{1}{2}=5\text{min}$.

$\therefore$ Total time = 3 + 5 = 8 min.

8. (c) 1 minute's work of each of the three pipes

$$= \frac{1}{20}+\frac{1}{30}-\frac{1}{15}=\frac{3+2-4}{60}=\frac{1}{60}$$

i.e., work of 3 pipes for 3 minutes $= \frac{1}{60}$

$\therefore$ Work of 3 pipes for 55 minutes each

$$= \frac{1}{60}\times55 = \frac{11}{12}$$

$\therefore$ Remaining part to be filled

$$= 1-\frac{11}{12}=\frac{1}{12}$$

Now, pipe A will fill $\frac{1}{20}$ of the cistern in next 1 minute.

$\therefore$ Remaining portion to be filled by pipe B

$$= \frac{1}{12} - \frac{1}{20} = \frac{5-3}{60} = \frac{2}{60} = \frac{1}{30}$$

$\therefore$ Time taken by pipe B to fill $\frac{1}{30}$ of the cistern

$$= \frac{1}{30} \times 30 = 1\text{min}$$

Hence, total time = (55 × 3) + 1 + 1 = 167 min.

9. (a) Let cistern will be full in x min. Then,

part filled by B in x min + part filled by A in (x – 4) min = 1

$$\Rightarrow \frac{x}{16} + \frac{x-4}{12} = 1$$

$$\Rightarrow x = \frac{64}{7} = 9\frac{1}{7}\text{hours.}$$

10. (a) Let A was turned off after x min. Then,

cistern filled by A in x min + cistern filled by B in (x + 23) min = 1

$$\Rightarrow \frac{x}{45} + \frac{x+23}{40} = 1$$

$$\Rightarrow 17x + 207 = 360 \Rightarrow x = 9\text{min.}$$

11. (a) Let cistern will be full in x min. Then,

part filled by A in x min + part filled by B in (x – 1) min + part filled by C in (x – 2)min = 1

$$\Rightarrow \frac{x}{3} + \frac{x-1}{4} + \frac{x-2}{6} = 1$$

$$\Rightarrow 9x = 19 \Rightarrow x = \frac{19}{9} = 2\frac{1}{9}\text{min}$$

12. (b) Total number of pipes = 6 (given)

Let number of inlet pipes = x

$\therefore$ number of outlet pipes = 6 –x

Now, Inlet pipe fill the tank in 9 hours and outlet pipe empty it in 6 hours.

$\therefore$ Total part filled in 1 hour = $\frac{x}{9} - \frac{6-x}{6}$

When all the pipes are opened.

But given total part filled in 9 hr

$$\therefore \quad \frac{x}{9} - \frac{6-x}{6} = \frac{1}{9} \Rightarrow 6x - 54 + 9x = \frac{54}{9} = 6$$

$$\Rightarrow \quad 15x = 60 \Rightarrow x = 4$$

Hence, number of inlet pipes = 4.

13. (a) Part filled in 10 hours

$$= 10\left(\frac{1}{15} + \frac{1}{20} - \frac{1}{25}\right) = \frac{23}{30}.$$

Remaining part = $\left(1 - \frac{23}{30}\right) = \frac{7}{30}$.

(A + B)'s 1 hour's work = $\left(\frac{1}{15} + \frac{1}{20}\right) = \frac{7}{60}$.

$$\frac{7}{60} : \frac{7}{30} :: 1 : x$$

or $x = \left(\frac{7}{30} \times 1 \times \frac{60}{7}\right) = 2$ hours.

$\therefore$ The tank will be full in (10 + 2) hrs = 12 hrs.

14. (c) (A + B)'s 1 hour's work

$$= \left(\frac{1}{12} + \frac{1}{15}\right) = \frac{9}{60} = \frac{3}{20}$$

(A + C)'s 1 hour's work

$$= \left(\frac{1}{12} + \frac{1}{20}\right) = \frac{8}{60} = \frac{2}{15}$$

Part filled in 2 hrs

$$= \left(\frac{3}{20} + \frac{2}{15}\right) = \frac{17}{60}$$

Part filled in 6 hrs

$$= \left(3 \times \frac{17}{60}\right) = \frac{17}{20}$$

Remaining part

$$= \left(1 - \frac{17}{20}\right) = \frac{3}{20}$$

Now, it is the turn of A and B and $\frac{3}{20}$ part is filled by A and B in 1 hour.

$\therefore$ Total time taken to fill the tank

= (6 + 1) hrs = 7 hrs.

15. (b) Part filled by (A + B + C) in 3 minutes

$$= 3\left(\frac{1}{30}+\frac{1}{20}+\frac{1}{10}\right)=\left(3\times\frac{11}{60}\right)=\frac{11}{20}$$

Part filled by C in 3 minutes = $\frac{3}{10}$

$\therefore$ Required ratio = $\left(\frac{3}{10}\times\frac{20}{11}\right)=\frac{6}{11}$

16. (b) Part of tank filled in 1 hour = 1/3

Part of tank emptied in the same time

$$= \frac{1}{3}-\frac{1}{3.5}$$

Total time required to empty it

$$= \frac{1}{\frac{1}{3}-\frac{1}{3.5}} = 21 \text{ hours}$$

17. (d). Pipe 1 (Hot) $\rightarrow$ 3 + X, X $\rightarrow$ Pipe 2 (cold)

Together $\frac{X(X+3)}{2X+3}=6\frac{2}{3}\text{min.}$

$$= \frac{X(X+3)}{2X+3}=6\frac{2}{3}\text{min.}=\frac{20}{3}$$

$40X + 60 = 3X (X + 3)$

$\Rightarrow 40X + 60 = 3X^2 + 9X$

$\Rightarrow 3X^2 - 31X - 60 = 0$

$\Rightarrow$ X = 12 minutes

18. (c) In 1 hour, empty part = $\frac{1}{8}$th.

When tap is turned on, then

empty part in 1 hour = $\frac{1}{12}$th.

$\therefore$ Part of cistern emptied, due to leakage in

1 hour $= \frac{1}{8}-\frac{1}{12}=\frac{3-2}{24}=\frac{1}{24}$th

Now, In 1 min, cistern fill = 6 lit

$\therefore$ In $\frac{1}{60}$hr, cistern fill = 6 lit.

$\therefore$ Cistern can hold = $6 \times 60 \times 24$ litre = 8640 litre.

19. (d) Suppose the first pipe was closed after x hrs .

Then, first 's x hrs' supply + second's 16 hrs' supply = 1

or, $\frac{x}{24}+\frac{16}{32}=1$

$\therefore$ $\frac{x}{24} = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore$ x = 12 hrs.

Examination method :

The first pipe should work for

$\left(1-\frac{16}{32}\right)\times 24$ hrs. = 12hrs

20. (b) Let the faster pipe fills the tank in x hrs.

Then the slower pipe fills the tank in x + 10 hrs.

When both of then are opened, the reservoir will be filled in $\frac{x(x+10)}{x+(x+10)}$ = 12

Or, $x^2 - 14x - 120 = 0$ $\therefore x = 20, -6$

But x can't be – ve, hence the faster pipe will fill the reservoir in 20 hrs.

21. (a) It is clear from the question that the filler pipe fills the tank in 8 hrs and if both the filler and the leak work together, the tank is filled in 8 hrs . Therefore the leak will empty the tank in

$\frac{8\times10}{10-8}$ = 40 hrs.

22. (d) Work done by both the taps in 5 min.

$$= 5\left(\frac{1}{20}+\frac{1}{25}\right)=\left(5\times\frac{9}{100}\right)=\frac{9}{20}$$

Remaining part = $\left(1-\frac{9}{10}\right)=\frac{11}{20}$

Now, $\frac{1}{20}$ part is filled in 1 min.

So, $\frac{11}{20}$ part will be filled in 11 min.

Hence, the tank will be full in 11 min. more the cistern.

EXERCISE 3

1. (a) Let it takes t minutes to completely fill the tank.

Now, $\frac{t}{6}+\frac{t}{8}+\frac{t-6}{12}=1$

or $\frac{4t+3t+2t-12}{24}=1$

or $9t - 12 = 24$

or $9t = 36$

or $t=\frac{36}{9}=4$ min.

$\therefore$ $\frac{2}{3}$rd of the tank can be emptied in

$\frac{2\times 30}{3}$ = 20 min

2. (b) Both the pipes A and B can fill $\frac{1}{12}+\frac{1}{16}=\frac{7}{48}$ of the cistern in one minute, when their is no obstruction.

With obstruction, both the pipes can fill

$\frac{1}{12}\times\frac{7}{8}+\frac{1}{16}\times\frac{5}{6}=\frac{7}{96}+\frac{5}{96}=\frac{1}{8}$ of the cistern in one minute.

Let the obstructions were suddenly removed after x minutes.

$\therefore$ With obstruction, $\frac{x}{8}$ of the cistern could be filled in x minutes and so the remaining $1-\frac{x}{8}$ of the cistern was filled without obstruction in 3 minutes,

i.e. In one minute, $\frac{8-x}{24}$ of the cistern was filled with obstruction.

$\Rightarrow \frac{8-x}{24}=\frac{7}{48}\Rightarrow 16-2x=7$

$\Rightarrow$ x = 4.5 min.

3. (b) The pipes A and B together can fill

$\frac{1}{20}+\frac{1}{30}=\frac{1}{12}$ of the tank in one hour.

$\therefore$ $\frac{1}{3}$ of the tank is filled by both the pipes A and B together in 4 hours. ...(1)

Now because of developing a leak after 4 hours, both the pipes can fill $\frac{1}{12}-\frac{1}{36}=\frac{1}{18}$ of the tank in one hour [Becauses $\frac{1}{3}$rd of the water supplied by both the pipes goes out]

$\therefore$ Remaining $\frac{2}{3}$ of the tank can be filled by both the pipes in $\frac{2}{3}\times$ 18 = 12 hours ... (2)

$\therefore$ The total time taken to fill the tank is 16 hours.

4. (b) Part of tank filled in one minute in given condition

$\frac{1}{15}+\frac{1}{12}-\frac{1}{20}=\frac{4+5-3}{60}=\frac{6}{60}=\frac{1}{10}$

$\therefore$ Tank will be completely filled in 10 min.

5. (a) Work done by 3rd tap in 1 min

$=\frac{1}{15}-\left(\frac{1}{10}+\frac{1}{12}\right)=\frac{-7}{60}$part

–ve sign denotes that 3rd tap empty the tank.

Since, 3rd tap empty $\frac{7}{60}$part of the tank in 1 min.

$\therefore$ 3rd tap empty the full tank in $\frac{60}{7}$min. or

$=8\frac{4}{7}$min $\approx$ 8 min. 34 seconds

6. (c) Volume of the cone

$= \frac{1}{3} \times \pi R^2 H = 1635.4\ m^3$

$\Rightarrow$ Rate of water flow

$= \frac{1635.4}{4} = 408\ m^3/hr$

7. (a) The two filler tap can fill the

$\left(\frac{1}{20}+\frac{1}{30}\right)$ or $\frac{1}{12}$ part of tank in 1 min.

$\therefore$ The two filler tap can fill the tank in 12 min.

$\therefore$ Half of the tank will be filled in 6 min.

Hence, it took (24 – 6 = 18 min.) to fill the remaining half of the tank when the outlet pump is opened. Thus, the total time required to empty half of the tank

$= \frac{18\times 6}{18-6} = \frac{18\times 6}{12} = 9$ minutes

Thus, capacity of the tank

$= 100 \times 9 \times 2 = 1800$ litres

8. (c) Time taken by one tap to fill the cistern = $\frac{1}{10}$hr

and second tap fills the cistern = $\frac{1}{15}$hr

The time taken by the both tap to fill the cistern

$= \frac{1}{10}+\frac{1}{15} = \frac{3+2}{30} = \frac{5}{30} = \frac{1}{6}$

Thus, both tap fill the cistern in 6 minutes. Now, given when waste pipe is open, both can fill the cistern in $\frac{1}{18}$hr.

Time taken by waste pipe to empty the cistern

$= \frac{1}{6}-\frac{1}{18}$ hrs

$= \frac{3-1}{18} = \frac{2}{18} = \frac{1}{9}$ minutes

Hence, in 9 minutes waste pipe can empty the cistern.

9. (b) $\because$ cistern fill in 6 hours.

$\therefore$ in 1 hour, filled part $= \frac{1}{6}$th

Now, due to leakage, filled part in 1 hour $= \frac{1}{8}$th

Part of the cistern emptied, due to leakage in 1 hour

$= \frac{1}{6}-\frac{1}{8} = \frac{1}{24}$th

$\therefore$ The leakage will empty the full cistern in 24 hrs.

10. (c) Let B can fill the cistern in x min. Then,

then A can fill the cistern in $\frac{x}{3}$min

Given $x-\frac{x}{3} = 10 \Rightarrow x = 15$min

11. (b) Cistern filled by both pipes in one hour

$= \frac{1}{14}+\frac{1}{16} = \frac{15}{112}$th

$\therefore$ Both pipes filled the cistern in $\frac{112}{15}$hrs.

Now, due to leakage both pipes filled the cistern in

$\frac{112}{15}+\frac{32}{60} = 8$hrs.

$\therefore$ Due to leakage, filled part in one hour $= \frac{1}{8}$

$\therefore$ part of cistern emptied, due to leakage in one hour

$= \frac{15}{112}-\frac{1}{8} = \frac{1}{112}$th

$\therefore$ In 112hr, the leakage would empty the cistern.

12. (c) Let the required time be x hours, then

$\frac{1}{12}\left(\frac{3}{4}x\right)+\frac{1}{15}\left(x-\frac{3}{4}x\right)+\frac{1}{12}\left(x-\frac{3}{4}x\right) = 1$

$\Rightarrow \frac{x}{16}+\frac{x}{60}+\frac{x}{48} = 1$

$\Rightarrow$ x = 10 hours

13. (a) In one min, (A + B) fill the cistern

$= \frac{1}{12}+\frac{1}{18}=\frac{5}{36}$th

In 4 min, (A + B) fill the cistern

$= \frac{5}{36}\times 4=\frac{5}{9}$th

Rest part $=1-\frac{5}{9}=\frac{4}{9}$th

$\because \frac{1}{18}$th part is filled by B in one min.

$\therefore \frac{4}{9}$th part is filled by B in $18\times\frac{4}{9}=8$min.

14. (a) Part filled in 7 min.

$= 7\times\left(\frac{1}{36}+\frac{1}{45}\right)=\frac{7}{20}$

Remaining part $=\left(1-\frac{7}{20}\right)=\frac{13}{20}$

Part filled by (A + B + C) in 1 min.

$=\left(\frac{1}{36}+\frac{1}{45}-\frac{1}{30}\right)=\frac{1}{60}$.

15. (b) Tap A fills 4 buckets (4 × 5 = 20 litres) in 24 min.

In 1 hour tap A fills $\frac{20}{24}\times 60=50$ litres

In 1 hour tap B fills $= 8\times 5=40$ litres

In 1 hour tap C fills $\frac{2\times 5}{20}\times 60=30$litres

If they open together they would fill

$50+40+30=120$ litres in one hour

but full tank is emptied in 2 hours

So, tank capacity would be 120 × 2 = 240 litres.

16. (a) Let the filling capacity of pump be x m^3/min.

Then, emptying capacity of pump = (x + 10) m^3 /min.

$\therefore \frac{2400}{x}-\frac{2400}{x+10}=8$

$\Rightarrow x^2 + 10x - 3000 = 0$

$\Rightarrow (x - 50)(x + 60) = 0$

$\Rightarrow x = 50$ m^3/min.

17. (d) Since, flow of waste pipe = flow of filling pipe.

$\Rightarrow$ Filled part in one min = emptied part in one min.

$\therefore$ After opening the waste pipe for 2 min, cistern will be full in (5 + 2) = 7 min.

18. (c) Let the time be t hours after 1 a.m.

$\therefore \frac{t}{4}+\frac{(t-1)}{5}-\frac{(t-2)}{2}=1$

19. (a) Radius of the pipe (r) = 4 cm. = 0.04 meter

Volume of water flowing out per sec

$= \pi r^2 \times$ rate of flow

$= \frac{22}{7}\times 0.04^2\times 3$ cu meters $= 0.0151$ cubic m

Time taken to fill the tank

$= 40 \times 30 \times \frac{8}{0.0151}$ sec

$= \frac{40\times 30\times 8}{0.01}\times\frac{1}{3600}$hours $= 176.6$ hours

20. (c) Let h be the length of water column discharged in 1 hour or 1 minute.

Volume discharged by the 4 pipes = Volume discharged by the single pipe.

$4 \times \pi \times (1.5)^2 \times h$

$= \pi \times (r)^2 \times h$

$\therefore \quad r^2 = 9$

$\therefore \quad r = 3$, Diameter = 6 cm.

21. (c) Part filled by the inlet pipe in 1 hour

$=\left(\frac{1}{10}-\frac{1}{16}\right)=\frac{6}{160}=\frac{3}{80}$

Part filled by the inlet pipe in 1 minute

$=\frac{3}{80\times 60}=\frac{1}{1600}$

$\therefore$ Capacity of tank = 1600 × 8 = 12800 litres.